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Distributed asynchronous supply coordination for energy producers embedded in the energy grids

Desti Alkano, Jacquelin M.A. Scherpen, and Ming Cao

Abstract—This paper studies the congestion control and energy flow allocation of renewable energy producers equipped with local energy storage devices and energy converters. The producers are embedded in the existing energy grids. Based on the producers' own measurements and some coordination with the grid operators, the energy producers adjust locally their supply levels injected to the energy grids so as to maximize their profit without exceeding the grid capacities. We incorporate an asynchronous implementation in the distributed supply coordination and prove its convergence. We implement the proposed algorithm for Power-to-Gas facilities embedded in the energy grids, which consist of a gas grid, mobility sector, and power grid, to demonstrate that the distributed asynchronous supply coordination achieves the same optimal performances as those of the synchronized distributed supply coordination.

I. INTRODUCTION

The energy infrastructure is changing towards a multi-producer multi-consumer market due to the integration of renewable energy into the existing energy grids [1]. Integration of the gas, electricity and heat grids are foreseen, though most of the current research on how to optimize the grids and the embedding of renewables in the grids deals with a single grid perspective. Renewable energy producers typically produce their energy products from an intermittent source that fluctuates in terms of availability; for example, the owner of an anaerobic digester produces biogas from agricultural waste whose volume depends on agricultural seasons. The produced biogas can be converted to heat and power using a micro combined heat and power (μ -CHP) device before selling it to a power grid or it can be upgraded to green gas before selling the biogas to a low pressure gas grid [2]. In addition, a Power-to-Gas facility can generate gaseous energy carrier, i.e. hydrogen, from excess power output of solar panels and wind turbines that highly depend on the weather conditions [3]. The hydrogen can then be injected to a gas grid, sold to a mobility industry, or reconverted into electrical energy using a fuel cell before selling it to a power grid at a later moment. Thus, for embedding of biogases, μ -CHP, and Power-to-Gas facilities, both grid and production capacities play an important role.

Given the increased expectations of the renewable energy producers on creating as much profit as possible from their

energy products and the limited grid capacities, the reliability of the energy grids has been put in danger. Strictly speaking, overloading grids result in severe consequences for the grid stability. It is therefore evident that the need for congestion control and energy flow control will increase as the penetration of renewable energy increases.

In this paper, we deal with both the congestion control and energy flow allocation problem to maximize the aggregated profit of all energy producers without exceeding grid capacities. When the optimization problem is solved centrally, it requires the knowledge of all profit functions of energy producers and complex coordination among them because of their coupling through the limited capacity in the existing grids. Both the competitive nature of energy producers and geographical embedding of renewables resulting in a large scale and highly complex grid give rise to the challenge that successfully solving the optimization problem centrally is highly unlikely. We therefore propose to solve the problem in a distributed fashion, in which each energy producer locally maximizes its profit, based on its local information yet some coordination with the operator of the grids is still necessary to avoid overloading grids.

In practice, energy producers and the operators may not have a common clock for their updates. Moreover, they may be located in different areas with different distances resulting in a substantial communication delay. We therefore include an asynchronous implementation in our proposed distributed supply coordination.

There has been a sustained effort on the resource allocation problem over the years to develop algorithms for distributed supply coordination. Inspired by [4], recent papers such as [5]-[8] have presented distributed algorithms for agents sharing the same network with limited capacity. These algorithms optimize resource utilization and take into account fairness among competing sources in order to handle the fact that the owner of the sources tends to supply as much source as possible to maximize his profit disregarding the mutually caused interference. These works rely on a dual decomposition approach combined with gradient projection methods in order to provide a distributive solution for each agent in the network. The implementation of asynchronous updates in the algorithms has been treated in [4],[7], and [8]. These studies were restricted to the case when each agent has only one controllable input but has freedom to choose paths to deliver its input to a destination.

Unlike [4], [7], and [8], in this paper we consider the case when each agent, i.e. energy producer, has more than one controllable input. The inputs are energy products, such as

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renewable gas and renewable power, as the energy producers are equipped with energy converters. Each energy product is delivered to a particular grid that has a limited capacity. This delivery process is through a single transmission line. In other words, there is no freedom for energy producers to choose paths to deliver their energy product to a particular grid. Our problem formulation seems similar to the problem shown in [9]. However, the utility functions, which are the profit functions in our model, presented in [9] are not separable for each product, yet the asynchronous implementation is not considered in [9]. Furthermore, we assume that each producer has a local energy storage device to have more freedom in utilizing its produced energy. It requires us to take the dynamics of the available energy in the storage device into account. Assuming that there is sufficient source to produce energy products, that the profit functions of energy producers are increasing, strictly concave, and twice differentiable and that the consecutive updates done by energy producers and the grid operators are bounded, for a fixed time (hence statically) we prove that our distributed asynchronous supply coordination converges to the same optimal values as the distributed synchronous supply coordination.

The paper is structured as follows. We specify the problem formulation and assumptions in Section II. We then propose a distributed asynchronous supply coordination for each energy producer and each grid operator in Section III. The Power-to-Gas case study is presented in Section IV to implement the algorithm shown in Section III for Power-to-Gas facilities embedded in the same gas grid, the same mobility industry, and the same power grid. Discussion and the convergence proof of the proposed algorithm are presented in Section V and Appendix, respectively.

II. PROBLEM FORMULATION

In this section, we develop a model for the supply coordination in a multi-producer grid. The goal is to maximize the estimated profit for all producers, without exceeding grid capacities. This goal corresponds to adjusting producers' supply levels. We further discuss assumptions we use in our convergence analysis.

We consider a setup with a set $I = \{1, \dots, n\}$ of agents, being renewable energy producers equipped with local storage devices. Each agent i produces intermittent source at a level of $h_i(k)$ at a time instance $k \in \{1, \dots, K\}$. From the source, agent i generates a set $M = \{1, \dots, q\}$ of energy products. Each product $x_{m,i}(k)$ of agent i is supplied to a particular grid, which has a limited capacity $c_m(k)$. We here assume that we have 100 % efficiency of the source converting to the energy products. The supply level of product $x_{m,i}(k)$ is characterized by $(U_{m,i}(k), \mathcal{X}_{m,i})$. Agent i receives a profit $U_{m,i}(k)$ when it supplies at a level of $x_{m,i}(k)$, that satisfies transmission bounds $\mathcal{X}_{m,i} = \{x_{m,i}(k) | \underline{x}_{m,i} \leq x_{m,i}(k) \leq \bar{x}_{m,i}\}$. The profit is modeled as a utility function borrowed from the microeconomics discipline [10].

When the intermittent source $h_i(k)$ exceeds the total energy production $\sum_{m=1}^q x_{m,i}(k)$, agent i stores the surplus

source $h_i(k) - \sum_{m=1}^q x_{m,i}(k)$ in its local storage device. Otherwise, agent i discharges some source from the device to produce energy products. Assuming that there is no leakage of the stored source over time, the dynamics of available source $z_i(k)$ in the storage device is given by

$$z_i(k+1) = z_i(k) + \rho \left(h_i(k) - \sum_{m=1}^q x_{m,i}(k) \right), \quad (1)$$

where ρ is the efficiency of (dis)charging the storage device. Suppose that the maximum capacity of the storage device is denoted by $S_i(k)$, the available source in the storage device is limited by

$$0 \leq z_i(k) \leq S_i(k). \quad (2)$$

When there is no more remaining space for the surplus source, the excess source needs to be flared. We here assume that there is no cost associated with the flaring process.

As briefly stated earlier, the control objective is to decide the supply levels $x_{m,i}(k)$ for all agents $i \in I$ and all products $m \in M$ so as to:

$$\max_{x_{m,i}} \sum_{i=1}^n \sum_{m=1}^q U_{m,i}(k) \quad (3)$$

subject to (1)-(2) and

$$\sum_{i=1}^n x_{m,i}(k) \leq c_m(k) \quad \forall m = 1, \dots, q. \quad (4)$$

The coupling constraints (4) ensure that the aggregate supply level of each energy product m from all $i \in I$ does not exceed the capacity of associated grid $c_m(k)$.

We further denote the total profit of agent i as $U_i(k) = \sum_{m=1}^q U_{m,i}(k)$. We use the following assumptions on the total profit functions $U_i(k)$.

- C1 On interval $\mathcal{X}_{m,i}$ for all $m \in M$, the total profit functions U_i are increasing, strictly concave, and twice continuously differentiable.
- C2 The curvatures of $U_i(k)$ are bounded away from zero for all $x_{m,i} \in \mathcal{X}_{m,i}$ with $m = 1, \dots, q$: $-U_i''(k) \geq \begin{pmatrix} \frac{1}{\alpha_{1i}} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1}{\alpha_{qi}} \end{pmatrix} > 0$.

Under C1, each agent i only sells energy product $x_{m,i}(k)$ when it gains a utility $U_{m,i}(k)$. There exists a unique maximizer $x_{m,i}(k)$ at most, for all $i \in I$ and $m \in M$, as the objective function $U_i(k)$ follows the conditions C1 and C2 and all constraints are compact and convex [11].

The objective function (3) is separable for each agent i and for each energy product m . However, the constraints (4) make agents coupled with each other. Hence, solving the optimization problem (3) requires coordination among agents. Due to the competitive nature of energy producers, it is highly unlikely that agents are willing to share all

information, including their states, to others. Hence, we solve the problem (3) in a distributed manner in which each agent i can make a decision on their supply levels $x_{m,i}(k)$ for all $m \in M$, based on their local information and some coordination with the grid operators to avoid overloading grids. Practically, the agents and the grid operators may not have a common clock to synchronize their updates. We therefore implement an asynchronous setting when solving the problem (3) in a distributed fashion, as presented in the following section.

III. DISTRIBUTED ASYNCHRONOUS SUPPLY COORDINATION

In this section, we develop a distributed asynchronous algorithm to optimally adjust the supply levels $x_{m,i}(k)$, given the limited grid capacities $c_m(k)$, intermittent source $h_i(k)$, and the available energy in the storage device $z_i(k)$. We propose the algorithm in a static environment, assuming that there is sufficient source to produce energy products. With this environment, we only consider the problem of a particular time instance k , with $\sum_{m=1}^q x_{m,i} \leq h_i + z_i \leq \sum_{m=1}^q \bar{x}_{m,i}$.

To develop the algorithm, we start with decomposing the problem (3) for each agent i using a dual decomposition method. We apply gradient iterations and stopping criterion to reach a consensus between agents and grid operators.

Let us define the Lagrangian function $L(x_{1,i}, \dots, x_{q,n}, \lambda_1, \dots, \lambda_q)$ of the problem (3) by $L(x_{1,i}, \dots, x_{q,n}, \lambda_1, \dots, \lambda_q) = \sum_{i=1}^n U_i - \sum_{m=1}^q \lambda_m (\sum_{i=1}^n x_{m,i} - c_m)$, where λ_m is the associated dual variables of coupling constraints (4). The objective function of the dual problem is therefore given by

$$\begin{aligned} D(\lambda) &= \max_{x_{1,i}, \dots, x_{q,n}} L(x_{1,i}, \dots, x_{q,n}, \lambda_1, \dots, \lambda_q) \\ &= \sum_{i=1}^n W_i(\lambda) + \sum_{m=1}^q c_m \lambda_m, \end{aligned} \quad (5)$$

where

$$W_i(\lambda) = \max_{x_{1,i}, \dots, x_{q,i}} U_i(k) - \sum_{m=1}^q \lambda_m x_{m,i}, \quad (6)$$

representing the exact profit function of each agent i , given dual variables λ . Hence, we obtain the dual problem specified by

$$\min_{\lambda \geq 0} D(\lambda) \quad \forall m = 1, \dots, q. \quad (7)$$

In what follows, we introduce an algorithm for each agent i and operator of the energy grid m . It implements asynchronous iterative search for optimal supply levels $x_{m,i}$ and dual variables λ_m , respectively. The algorithm is inspired by [4]. The authors in [4] considered a single product for each agent i , while here we take into account q energy products which correspond to q different utility functions for each agent i .

A. Algorithm for an operator of the energy grid m .

Within the time interval $[k, k+1]$, let $R_m \subseteq \{1, 2, \dots\}$ be a set of internal times at which the operator of an energy grid m adjusts its dual variable λ_m based on its current knowledge of aggregated supply levels from all agents $i = 1, \dots, n$. At times $r \in R_m$, the operator m computes an estimate $l_m(r)$ by

$$\begin{aligned} l_m(r) &= c_m - \sum_{i=1}^n \hat{x}_{m,i}(r) \\ &= c_m - \sum_{i=1}^n \sum_{r'=r-r_o}^r a_{m,i}(r', r) \cdot x_{m,i}(r'), \end{aligned} \quad (8)$$

with $\sum_{r'=r-r_o}^r a_{m,i}(r', r) = 1 \quad \forall r$ representing the weighting factor of total supply bids received by the operator m . It then updates its dual variable $\lambda_m(r)$ based on $\lambda_m(r+1) = [\lambda_m(r) - \gamma l_m(r)]^+$ where γ is a sufficiently small step size. At times $r \notin R_m$, the dual variable $\lambda_m(r)$ is unchanged, resulting in $\lambda_m(r+1) = \lambda_m(r)$, $r \notin R_m$. The operator m terminates the updates of its dual variable $\lambda_m(r)$ when the difference between its consecutive updates $\lambda_m(r)$ is within a bound ξ_m and when its estimated total supply bids are below its grid capacity.

Remark 1 The dual variable $\lambda_m(r)$ experienced by the operator m at time r can be interpreted as an additional cost, namely distribution charge, when the operator detects an overloading. As we have the constraint $\lambda_m \geq 0$, it does not provide any incentive when the total supply is below the capacity of the energy grid m .

B. Algorithm for each agent i

Assume that there is sufficient source for an agent i to produce energy products at each time instant k , given by $\sum_{m=1}^q x_{m,i}(k) \leq h_i(k) + z_i(k) \leq \sum_{m=1}^q \bar{x}_{m,i}(k)$. Let $R_i \subseteq \{1, 2, \dots\}$ be a set of internal times within the time $[k, k+1]$ at which the agent i updates its supply level $x_{m,i}(r)$ for all $m \in M$ by optimizing (6). At times $r \in R_i$, the agent i calculates estimates of distribution charges $\hat{\lambda}_m(r)$ for all $m \in M$ by $\hat{\lambda}_m(r) = \sum_{r'=r-r_o}^r b_{m,i}(r', r) \cdot \lambda_m(r')$ with $\sum_{r'=r-r_o}^r b_{m,i}(r', r) = 1$ for all $r \in R_i$ which specify the weighting factor of distribution charges received by agent i . Based on the estimated distribution charges $\hat{\lambda}_m(r)$ for all $m \in M$, the agent i solves the optimization problem (6) and therefore obtains $x_{m,i}(r) = x_{m,i}(\hat{\lambda}_m(r))$. At times $r \notin R_i$, $x_{m,i}(r+1) = x_{m,i}(r)$ for all $m \in M$. Agent i terminates its iterations when the successive updates $x_{m,i}(r)$ are within a bound ξ_i .

We use the following assumption on the time between the successive updates.

C3 The time between the consecutive updates is bounded by r_o for both the updates of distribution charges and supply bids.

Remark 2 The problem (7) can be rewritten as $\min_{\lambda_1} \dots \min_{\lambda_q} D(\lambda_1, \dots, \lambda_q)$. When the agents attain higher profit by selling its energy product to a particular grid m than other grids, the agents will first search for the optimal value of its corresponding supply level. Strictly speaking, at each time r the agent i will solve one of its controllable inputs.

In this case, we can directly use the proof of Theorem 2 in [4] to establish the convergence of our proposed distributed asynchronous algorithm.

In this study, we aim at generalizing the case for all possible values of $U_{m,i}$. In particular, we deal with the case when all grids $m \in M$ give the same profit to agent i . Hence, the agent i has no priority in calculating $x_{m,i}(r)$ at each iteration time r . In other words, the agent i optimizes its controllable inputs $x_{m,i}(r)$ for all energy products $m \in M$ at the same time r .

Define the error in distribution charge estimation as $\Delta\lambda(r) = [\Delta\lambda_1(r), \dots, \Delta\lambda_q(r)]^T$ where $\Delta\lambda_m(r) = |\hat{\lambda}_m^i(r) - \lambda_m^i(r)|$, the deviation in supply level estimation as $\Delta x_i(r) = [\Delta x_{1,i}(r), \dots, \Delta x_{q,i}(r)]^T$ where $\Delta x_{m,i}(r) = |\hat{x}_{m,i}(r) - x_{m,i}(r)|$, and the error in gradient estimation as $\Delta l(r) = [\Delta l_1(r), \dots, \Delta l_q(r)]^T$ where $\Delta l_m(r) = |l_m(r) - \frac{\partial D}{\partial \lambda_m}(\lambda(r))|$.

The following theorem brings together assumptions C1-C3 to establish the convergence of the proposed distributed asynchronous supply coordination stated in subsections 3.A-3.B.

Theorem 1: Given the knowledge that $\sum_{m=1}^q \bar{x}_{m,i}(k) \leq h_i(k) + z_i(k) \leq \sum_{m=1}^q \bar{x}_{m,i}(k)$, any initial supply levels $x_{m,i}(0)$, any initial distribution charges $\lambda_m(0) \geq 0$ and suppose that assumption C1-C3 hold, the error in distribution charge estimation $\Delta\lambda(r)$, the deviation in supply level estimation $\Delta x_i(r)$, and the error in gradient estimation $\Delta l(r)$ all converge to zero as $r \rightarrow \infty$, for all $i \in I$ and $m \in M$.

Proof: See Appendix. \square

IV. POWER-TO-GAS CASE STUDY

Consider a Power-to-Gas (PtG) facility $i \in \{1, \dots, n\}$ equipped with an electrolyzer, a gas storage device, and a fuel cell. The facility is currently an alternative for existing energy storage options. The overview of PtG facility is shown in Fig. 1.

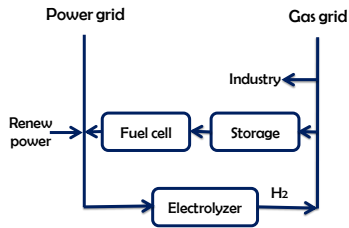


Fig. 1: An overview of the Power-to-Gas concept, adapted from [3].

The PtG facility uses the electrolyzer to produce a gaseous energy carrier, i.e. hydrogen, from excess power output. The excess power output results from the difference between the electricity usage and the power output from conventional power generator and the renewable energy source.

The hydrogen production level is denoted by $h_i(k) \in \mathbb{R}_+$, at each time instant $k \in \{1, \dots, K\}$. The PtG facility aims at creating profit from the produced hydrogen by injecting it into the gas grid and the mobility sector at a level of $g_i(k) \in \mathbb{R}_+$ and $y_i(k) \in \mathbb{R}_+$, respectively. Additionally, the produced hydrogen can be reconverted into electrical energy by the fuel cell before selling it to a power grid. This corresponding amount of hydrogen is denoted by $e_i(k) \in \mathbb{R}_+$. Clearly, $e_i(k) = 0$ if $h_i(k) > 0$.

Let us consider n PtG facilities embedded in the same gas grid, mobility sector, and power grid. We refer them to such a group of renewable energy producers as a community. The community aims at maximizing its total profit $\sum_{i=1}^n U_i(k)$, with $U_i(k)$ being the profit of each PtG facility i .

We model the profit $U_i(k)$ by $U_i(k) = \sum_{l=g,y,e} (p_l(k) - c_{l,i}(k))l_i(k) - c_{ql,i}l_i^2(k)$ where $p_l(k)$ is the selling price in an energy grid l , $c_{l,i}(k)$ is the associated cost in providing energy for an energy grid l , and $c_{ql,i}$ is the corresponding cost for transmission losses experienced by PtG facility i at time k . Inspired by [12], we define the cost of transmission losses in a quadratic form.

Due to capacities of transmission lines and pipelines, the energy injected to the gas grid, mobility sector, and power grid are limited by $\mathcal{L}_i = \{l_i(k) | \underline{l}_i(k) \leq l_i(k) \leq \bar{l}_i(k)\}$ for all $l = g, y, e$ and each k , with $\underline{l}_i(k)$ and $\bar{l}_i(k)$ are the lower and upper bounds of the energy injected in an energy grid l , respectively.

The PtG facilities are equipped with a pressured tank or cavern with the maximum capacity S_i . The dynamics of available hydrogen $z_i(k)$ in the storage device is given by (1) and the available hydrogen in the storage device is therefore limited by (2).

Regarding the grid capacities, there is a maximum allowable energy injected into the energy grid. We therefore have the following constraints.

$$\sum_{i=1}^n l_i(k) \leq c_l(k) \quad \forall l = g, y, e. \quad (9)$$

As briefly stated before, the objective of the community consisting of n PtG facilities embedded in the same gas grid, mobility sector, and power grid is to decide the supply levels $g_i(k), y_i(k), e_i(k)$ for all $i = 1, \dots, n$ so as to: $\max_{g_i, y_i, e_i} \sum_{i=1}^n U_i(k)$ subject to (1)-(2) and (9). We implement algorithms proposed in 3.A-3.B to provide a distributed asynchronous supply coordination for each PtG facility i and each grid operator m .

A. Simulation Results

We perform simulation studies for 5 PtG facilities embedded in the same gas grid, mobility sector, and power grid. We assume that all PtG facilities update their supply bids every 15 minutes, while the operators of the energy grids and the chemical sector do their updates each 30 minutes. The associated selling prices p_l and costs $c_{l,i}, c_{ql,i}$ of both energy grids and chemical sector are assumed to be identical, which are 0.2, 0.019, and 0.0001 €/Nm³, respectively. With

these uniform values for all $l = g, y, e$, the proposed algorithm calculates the optimal values of g_i, y_i, e_i at the same time. We set the hydrogen production level h_i for each PtG facility i at $5e+4 \text{ Nm}^3$ and the maximum capacity of its storage device is $10e+6 \text{ Nm}^3$. It implies no chance for the PtG facilities to inject their energy into the power grid. Moreover, we assume that there is no available hydrogen in their gas storage device. We set the capacity on the gas grid and the chemical sector at the same level, i.e. 43 Nm^3 .

We specify the step size γ at 0.0005. The stopping criterion ξ_l and ξ_i are set at $1e-6$ for all $l = g, y, e$ and $i = 1, \dots, 5$. The optimal solutions are calculated using QP-solver from Gurobi 5.6.3 embedded in MATLAB 2014a.

Fig. 2 shows the evolution of distribution charges and the total supply bids occurring in the gas grid and the mobility sector when $\lambda_g(0), \lambda_y(0), \lambda_e(0) = 1$. As shown in the figure, setting the initial distribution charges at one makes all PtG facilities not interested in injecting their hydrogen to the gas grid and the mobility sector. It is due to the fact that the selling prices ($p_g = p_y = 0.1 \text{ €/Nm}^3$) are much lower than the initial distribution charges, i.e. $\lambda_g(0), \lambda_y(0), \lambda_e(0) = 1$. Hence, the operators of the gas grid and the mobility sector iteratively reduce their distribution charges up till their grid capacities are satisfied by the PtG facilities. As expected, the convergence is much faster when the updates are done synchronously.

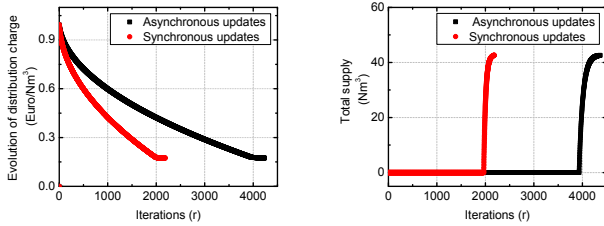


Fig. 2: The left figure shows the evolution of distribution charges in the gas grid and the mobility sector, whereas the right figure presents the total supply bids from 5 PtG facilities for the gas grid and the mobility sector. The distribution charges are initially set at one and the stopping criterion ξ_l, ξ_i are set at $1e-6$ for all $l = g, y, e$, and $i = 1, \dots, 5$.

V. DISCUSSION

In this paper, we have shown how the energy producers coordinate their supply bids to the grid operators so as to maximize their profit, subject to their technical constraints and the grid capacities. We considered an asynchronous implementation when solving the problem in a distributed manner. We established conditions under which we can guarantee the convergence of our distributed asynchronous supply coordination algorithm. The algorithm helps the energy producers to better allocate their produced energy and helps the operators to avoid overloading grids.

The algorithm was established for a static environment. As the current period's decision may affect future condition in the storage device, we are extending the algorithm by taking

into account the dynamics of available energy in the storage device when iteratively calculating the energy products.

APPENDIX: PROOF OF THEOREM 1

Lemma 1: Suppose that assumption C1 holds. The dual objective function $D(\lambda)$ is convex, lower bounded, and continuously differentiable.

Proof: The dual objective function $D(\lambda)$ is convex and lower bounded due to the properties of its primal objective function.

As the primal objective function is increasing and strictly concave while all constraints are convex, the duality gap does not exist. Hence, the Lagrangian function $L(x_{1,i}, \dots, x_{q,n}, \lambda_1, \dots, \lambda_q)$ has a unique maximizer for each controllable input $x_{m,i}$. From [[13], Prop. 6.1.1], it is known that $D(\lambda)$ is continuous differentiable if its Lagrangian function has a unique maximizer. \square

$$\text{We define } \beta_i(\lambda) = \begin{bmatrix} \beta_{1,i}(\lambda_1) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \beta_{q,i}(\lambda_q) \end{bmatrix} \text{ where}$$

$$\beta_{m,i}(\lambda_m) = \begin{cases} \frac{1}{-U_i'(\bar{x}_{m,i}(\lambda_m))} & \text{if } U_i'(\bar{x}_{m,i}) \leq \lambda_m^i \leq U_i'(x_{m,i}) \\ 0 & \text{otherwise,} \end{cases} \quad (10)$$

for all $m = 1, \dots, q$. Following from C2, for all $\lambda_m \geq 0$ we

$$\text{have } 0 \leq \beta_i(\lambda) \leq \alpha_i < \infty \text{ where } \alpha_i = \begin{bmatrix} \alpha_{1,i} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \alpha_{q,i} \end{bmatrix}.$$

Furthermore, let $B(\lambda)$ be the $q \times q$ matrix specified by

$$B(\lambda) = \begin{bmatrix} \sum_{i=1}^n \beta_{1,i} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sum_{i=1}^n \beta_{q,i} \end{bmatrix}.$$

Let $x_{m,i}(\lambda_m)$ be the unique maximizer in (6). According to KKT theorems, we have $\lambda_m = U_i'(x_{m,i}(\lambda_m))$ for all $m \in M$, where $U_i'(x_{m,i})$ denotes the derivatives of $U_i(x_{m,i})$. Hence, with $U_i'^{-1}$ are the inverse function of U_i' , we obtain $x_{m,i} = [U_i'^{-1}(\lambda_m)]_{x_{m,i}}^{\bar{x}_{m,i}}$, where $[j]_a^b = \min\{\max\{j, a\}, b\}$ are the projection on the interval a and b .

We use the following lemmas from [4] to prove Theorem 1.

Lemma 2: Suppose that assumption C1 holds. The Hessian of the dual objective function $D(\lambda)$ is given by $\nabla^2 D(\lambda) = B(\lambda)$, where it exists.

$$\text{Let } \pi(r) = \begin{bmatrix} \pi_1(r) \\ \vdots \\ \pi_q(r) \end{bmatrix} = \begin{bmatrix} \lambda_1(r+1) - \lambda_1(r) \\ \vdots \\ \lambda_q(r+1) - \lambda_q(r) \end{bmatrix}. \text{ Given any } \bar{\lambda} \in \mathbb{R}_+^{3 \times 1}, \text{ define } u_i(\cdot; \bar{\lambda}) : \mathbb{R}_+^{3 \times 1} \rightarrow \mathbb{R}_+ \text{ by } u_i(\varepsilon; \bar{\lambda}) = U_i'^{-1}(\varepsilon^T \bar{\lambda}) = U_i'^{-1}(\sum_r \varepsilon_r \bar{\lambda}_r).$$

$$\text{Assume all conditions C1-C3 hold and define } l(r) = \begin{bmatrix} l_1(r) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & l_q(r) \end{bmatrix}.$$

Lemma 3: For all r , $l(r)\pi(r) \leq -\frac{1}{\gamma}\pi^2(r)$, where $\pi^2(r) = [\pi_1^2(r), \dots, \pi_q^2(r)]^T$.

Lemma 4: There exists a constant $A_1 > 0$ such that, for all $\lambda \geq 0$ and all q , we have $q^T \nabla^2 D(\lambda) q \leq 4nA_1 \|q\|^2$.

Define the upper bound of $\sum_r \tilde{\lambda}_{m,r}$ as $\bar{\lambda}_{m,r}$, for all $m = 1, \dots, q$.

Lemma 5: For any distribution charge $\bar{\lambda}_m \geq 0$, it follows

$$0 \leq \begin{bmatrix} \frac{\partial u_i}{\partial \varepsilon_{1,r}(\varepsilon; \bar{\lambda}_1)} \\ \vdots \\ \frac{\partial u_i}{\partial \varepsilon_{q,r}(\varepsilon; \bar{\lambda}_q)} \end{bmatrix} \leq \alpha_i \begin{bmatrix} \bar{\lambda}_{1,r} \\ \vdots \\ \bar{\lambda}_{q,r} \end{bmatrix}.$$

Define now $U_i'^{-1}(\hat{\lambda}_m^i(r)) = u_i(\varepsilon_{m,r'}(r); \bar{\lambda}_m(r))$ and $U_i'^{-1}(\lambda_m^i(r)) = u_i(1_{m,r'}(r); \bar{\lambda}_m(r))$, where $\hat{\lambda}_m^i(r)$ and $\lambda_m^i(r)$ are the estimated and exact distribution charges for energy grid m , respectively, and $1_{m,r'}(r) = \begin{cases} 1 & \text{if } r' = r, \\ 0 & \text{otherwise,} \end{cases}$ and $\varepsilon_{m,r'}(r) = b_{m,i}(r', r)$ for all $m = 1, \dots, q$.

Define $\Delta U_i'^{-1}(\lambda(r)) = [\Delta U_i'^{-1}(\lambda_1(r)), \dots, \Delta U_i'^{-1}(\lambda_q(r))]^T$ where $\Delta U_i'^{-1}(\lambda_m(r)) = |U_i'^{-1}(\hat{\lambda}_m^i(r)) - U_i'^{-1}(\lambda_m^i(r))|$ and $\pi_{abs}(r') = [\pi_{abs,1}(r'), \dots, \pi_{abs,q}(r')]^T$ with $\pi_{abs,m}(r') = |\pi_m(r')|$.

Lemma 6: $\Delta U_i'^{-1}(\lambda(r)) \leq \alpha_i \sum_{r'=r-r_0}^{r-1} \pi_{abs}(r')$ for all r .

Define $\Lambda U_i'^{-1}(\lambda(r)) = [\Lambda U_i'^{-1}(\lambda_1(r)), \dots, \Lambda U_i'^{-1}(\lambda_q(r))]^T$ where $\Lambda U_i'^{-1}(\lambda_m(r)) = |U_i'^{-1}(\hat{\lambda}_m^i(r)) - U_i'^{-1}(\lambda_m^i(r))|$.

Lemma 7: $\Lambda U_i'^{-1}(\lambda(r)) \leq \alpha_i \sum_{r'=\tau}^{r-1} \pi_{abs}(r')$, for all r .

Lemma 8: There exists a constant $A_2 > 0$ such that $\Delta l(r) \leq nA_2 \bar{\alpha}_i \sum_{r'=r-2r_0}^{r-1} \pi_{abs}(r')$ for all r , where $\bar{\alpha}_i$ is the upper bound of α_i .

Lemma 9: Given step size γ which is sufficiently small, for all r we have $D(\lambda_m(r+1)) \leq D(\lambda_m(0)) - (\frac{1}{\gamma} - 2nA_1 - nA_2 \bar{\alpha}_{mi}) \sum_{\tau=0}^r |\pi_m(\tau)|^2$ for all $m = 1, \dots, q$.

As stated in assumption C1, we have the primal functions U_i which is increasing, strictly concave, and twice differentiable. Its dual function $D(\lambda)$ is therefore decreasing, convex, lower bounded, and continuously differentiable (Lemma 1). The optimal distribution charge λ_m^* is found when $\lim_{r \rightarrow \infty} D(\lambda_m(r)) - D(\lambda_m(r-1)) = 0$. Hence, from Lemma 9 we obtain

$$\lim_{r \rightarrow \infty} |\pi_m(r)| = 0. \quad (11)$$

We are now ready to prove Theorem 1. According to (11), for all agents $i = 1, \dots, n$ we have the estimated and exact distribution charges given by

$$\begin{aligned} |\hat{\lambda}_m(r) - \lambda_m(r)| &= \left| \sum_{r'=r-r_0}^r b_{mi}(r', r) \lambda_m(r') - \lambda_m(r) \right| \\ &\leq \max_{r-r_0 \leq r' \leq r} |\lambda_m(r') - \lambda_m(r)| \\ &\leq \max_{r-r_0 \leq r' \leq r} \sum_{\tau=r'}^{r-1} |\pi_m(\tau)| \leq \sum_{r'=r-r_0}^r |\pi_m(r')|. \end{aligned}$$

Therefore, $\lim_{r \rightarrow \infty} |\hat{\lambda}_m(r) - \lambda_m(r)| = 0$.

As we have defined the error in distribution charge estimation by $\Delta \lambda(r) = [\Delta \lambda_1(r), \dots, \Delta \lambda_q(r)]^T$ where $\Delta \lambda_m(r) =$

$|\hat{\lambda}_m^i(r) - \lambda_m^i(r)|$, we obtain $\lim_{r \rightarrow \infty} \Delta \lambda(r) = 0$.

We have the estimated value for controllable inputs $x_{m,i}(r)$ defined by $x_{m,i}(r) = x_{m,i}(\hat{\lambda}_m(r))$ where $x_{m,i}(\cdot) = [U_i'^{-1}(\cdot)]_{\bar{x}_{m,i}}^{\bar{x}_{m,i}}$, $\hat{\lambda}_m(r) = \sum_{r'=r-r_0}^r b_{mi}(r', r) \lambda_m(r')$, and $\sum_{r'=r-r_0}^r b_{mi}(r', r) = 1$ for all $m = 1, \dots, q$. Remind that $\bar{x}_{m,i}(r)$ is the value of controllable inputs $x_{m,i}(r)$ when agent i knows the true distribution charges $\lambda_m(r)$ at time r . Note that $x_{m,i}(r)$ is bounded by a lower bound $\underline{x}_{m,i}$ and upper bound $\bar{x}_{m,i}$. Hence, by projection theorem {[13], Prop. 2.1.3.c}, Lemma 6, and (11), we obtain

$$\begin{aligned} |x_{m,i}(r) - \hat{x}_{m,i}(r)| &\leq |U_i'^{-1}(\hat{\lambda}_m(r)) - U_i'^{-1}(\lambda_m(r))| \\ &\leq \alpha_m \sum_{r'=r-r_0}^{r-1} |\pi_m(r')|. \end{aligned}$$

As a result,

$$\lim_{r \rightarrow \infty} |x_{m,i}(r) - \hat{x}_{m,i}(r)| = 0 \forall m = 1, \dots, q \quad (12)$$

Previously, we defined the deviation in supply level estimation as $\Delta x_i(r) = [\Delta x_{1,i}(r), \dots, \Delta x_{q,i}(r)]^T$ where $\Delta x_{m,i}(r) = |\hat{x}_{m,i}(r) - x_{m,i}(r)|$. From (12), we have $\lim_{r \rightarrow \infty} \Delta x_i(r) = 0$.

From Lemma 8 and (11), we obtain $\lim_{r \rightarrow \infty} |l_m(r) - \frac{\partial D}{\partial \lambda_m}| = 0$ for all $m = 1, \dots, q$.

As defined before, the error in gradient estimation is defined by $\Delta l(r) = [\Delta l_1(r), \dots, \Delta l_q(r)]^T$ where $\Delta l_m(r) = |l_m(r) - \frac{\partial D}{\partial \lambda_m}(\lambda(r))|$. Hence, $\lim_{r \rightarrow \infty} \Delta l(r) = 0$.

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